TYPE 712: RADIANT SLAB WITH EMBEDDED PIPES (INTERFACES WITH TYPE 56)

General Description

This component is intended to model a radiant floor-heating slab, embedded in soil, and containing a number of fluid filled pipes. The heat transfer within the slab is assumed to be conductive only and moisture effects are not accounted for in the model. The model relies on a three-dimensional finite difference method, solving the resulting inter-dependent differential equations using an iterative approach. The user may define any number of pipes within the slab through An external data file containing information about the path that the pipes follow through the slab for each pipe. The slab is assumed to be embedded in the soil and the user may define bottom and/or perimeter insulation that extends below the slab if desired. Default numbers of maximum slab nodes, maximum pipe nodes and maximum number of pipes are set, but may be increased by modification of the Fortran code. This model differs from Type705 in that it does not extend the 3D grid into the soil that surrounds the slab. Instead, the zone temperature communicates (through the slab) with a ground surface temperature generated by the Kasuda [1] correlation.

This version of the radiant slab model is designed to be used in conjunction with the Type56 building model. If you wish to use the same model in conjunction with a different building model (one that computes a zone air temperature but does not calculate interior surface temperatures), you should use Type713 instead. For more detail on this distinction, please see section 1 of the detailed description below.

Nomenclature

m	[kg]	Mass
C_p	[kJ/kg.K]	Specific heat.
T	[ºC]	Temperature
t	[hr]	time
$\dot{Q}_{in}$	[kJ/hr]	The rate at which energy enters a fluid, soil or slab node.
$\dot{Q}_{out}$	[kJ/hr]	The rate at which energy exits a fluid, soil or slab node.
$\dot{m}$	[kg/hr]	Mass flow rate of fluid entering a fluid node.
$T_{fluid,i}$	[ºC]	The temperature of fluid in a given fluid node.
$\overline{T}_{x,y}$	[ºC]	The average temperature of a given slab node.
x	[m]	The distance along the length direction.
y	[m]	The distance along the width direction.
U	[kJ/hr.m^2.K]	The loss coefficient from the pipe
A	[m^2]	Area
R_{in}	[hr.m^2.K/kJ]	The thermal resistance between the fluid and the pipe.
h_{inside}	[kJ/hr.m^2.K]	The convection heat transfer coefficient on the inside of the pipe.
$A_{surfaceinner}$	[m^2]	The area of the pipe interior heat transfer surface
Nusselt		The Nusselt number for pipe inside convection

d_{pipes}	[m]	Pipe average diameter
$d_{pipes,i}$	[m]	Pipe inside diameter
$d_{pipes,0}$	[m]	Pipe outside diameter
Re		Reynold's number
Pr		Prandtl number
R_{pipe}	[hr.m^2.K/kJ]	The thermal resistance of the pipe material.
$R_{contact}$	[hr.m^2.K/kJ]	The thermal contact resistance between the pipe and surrounding soil or slab.
k	[kJ/m.K]	Thermal conductivity.
dx	[m]	The x direction dimension of a soil or slab node.
dy	[m]	The y direction dimension of a soil or slab node.
$T_{ambient}$	[ºC]	The temperature of a node's surroundings. For an indoor node, $T_{ambient}$ is the temperature of the zone. For an outdoor node, $T_{ambient}$ is the outdoor dry bulb temperature.
$T_{surface}$	[ºC]	The surface temperature of a node exposed to ambient conditions.
$T_{surroundings}$	[ºC]	The effective temperature with which a node radiatively transfers energy. For an indoor node, $T_{surroundings}$ is the surface temperature of zone walls. For an outdoor node, $T_{surroundings}$ is the effective sky temperature.
$h_{convective}$	[kJ/hr.m^2.K]	The convective heat transfer coefficient for the surface of a node.
σ	[W/m^2.K^4]	The Stefan-Boltzmann constant (5.67×10^{-8} W/m^2.K^4).
ε	[0..1]	The emissivity of a surface.
$Q_{ambient}$	[kJ/hr]	Additional radiation incident on a node.
T_{Type56}	[ºC]	Temperature of the interior (zone side) surface generated by Type56
$T_{interface}$	[ºC]	Temperature of the interface between the Type56 layer and the Type705 layer
$T_{SlabNode}$	[ºC]	Temperature of a slab node

Detailed Description

1. Appropriate Building Models

Simplified building models such as the lumped capacitance or degree-day models (Types 660, 80, and 12 in TRNSYS for example) lack two concepts on which this version of the radiant slab model depend. First, they do not contain the concept of a boundary wall and second, they do not perform the radiation exchange calculations that determine the appropriate interior surface temperatures of all walls, floors and ceilings. Users wishing to utilize a radiant floor model with these simplified building models should consider using the TESS Type713 model instead. This model (Type712) is intended to interface with detailed building models that calculate the inside surface temperature of the wall and take as an INPUT the outside surface temperature of the wall. The standard multi-zone building model in TRNSYS, Type56, is such a model.

By defining a wall as a boundary wall in Type56, one assumes that there is either a known temperature on the other (non-zone) side of the wall, or that the temperature on the non-zone side is identical to the temperature in the zone. Normally the boundary temperature specified is the air temperature of a node connected to the back surface of the wall through a pure resistance (such as an air temperature coupled to the back side of the wall through a convective resistance). However by specifying the convective

resistance of the back side of the boundary wall to zero, the Type56 zone model assumes that the boundary temperature specified is the back-side surface temperature of the boundary wall – which is exactly what one needs for the interface between the soil storage model and the Type56 zone model to work correctly. Figure 1 depicts the front and back side temperatures used by the combination of Type56 and this slab model.

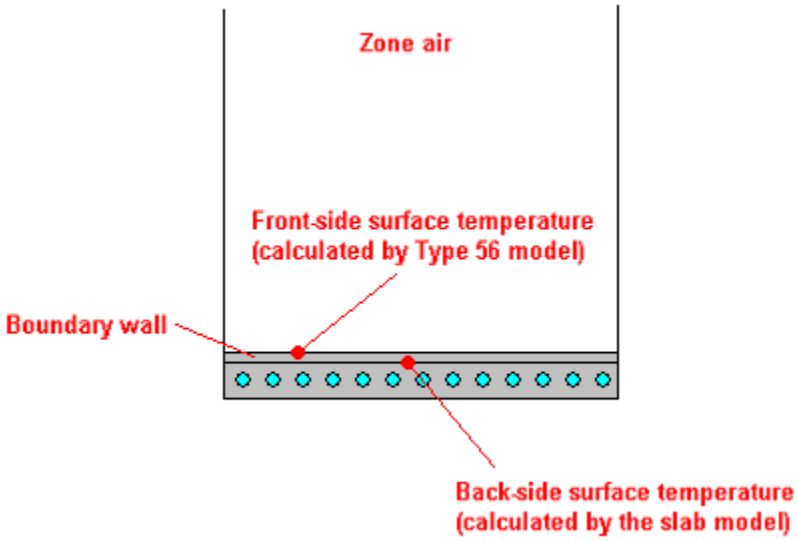


Figure 1: Boundary Wall Definition

To use Type712, one defines a portion of the slab floor in Prebid as a boundary wall with a "user defined" temperature as an input (as opposed to as a schedule or a constant value), and with a back-side heat transfer coefficient of zero. The boundary wall should contain all the layers of the slab above the layer that contains the fluid filled pipes (carpeting, pad, concrete etc.) The user then generates the Type56 files and checks the .inf results, entering the slab's U-value (calculated by Type56) in the appropriate Type712 parameter field. The U-value entered for the slab model should be for the slab only and should *not* contain the front or back-side convection coefficients. The user will also have to specify that one of the outputs from Type56 is the inside surface temperature of this boundary wall. The user then connects the surface temperature output from Type56 to the slab model's input and connects the first output of the slab model back to the boundary wall temperature required by Type56. Note that only a portion of the slab's thermal properties are specified in Type712; those of the layer that contains the fluid pipes. The thermal properties of the layers above the fluid pipe layer are accounted for within Type56 and by the definition of those layers as a boundary wall. In essence Type712 calculates the thermal behavior of the slab layer containing the fluid pipes. It takes the air/slab boundary temperature from Type56 and computes the slab/fluid layer boundary temperature required by Type56.

2. Slab and Surroundings

In Type712, the slab is broken into two parts. The first part is modeled by Type56 and should include whatever materials are in contact with the zone air. For example, this part might include carpet and pad, hard wood floor or a layer of concrete. Type56 will provide the zone-side surface temperature of this layer. The second part of the slab contains the fluid filled pipes. It's material is not limited but the layer should not be in contact with the zone air temperature. The layer will typically be concrete with

embedded pipes. Type712 computes the temperature of the interface between the layer modeled in Type56 as a boundary wall and the layer modeled by Type712. Figure 2 shows the layers schematically.

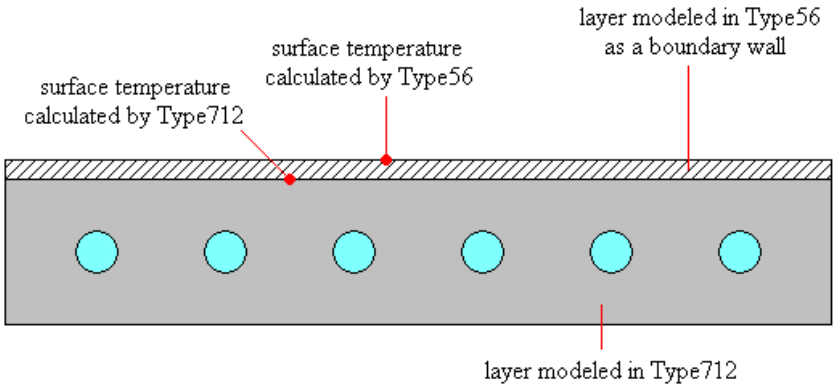


Figure 2: Division of Slab into Layer Modeled in Type56 and Layer modeled in Type712

If there is no layer in the slab that can be attributed to the Type56 boundary wall, a fictitious, purely resistive membrane layer must be added in Prebid so that Type56 is able to compute an interior surface temperature and so that there is an (all be it very small) temperature difference between the interior surface and the Type56/Type712 interface temperature.

Type712 further divides the slab layer containing fluid pipes into two parts, the slab and the far-field. First is the slab itself, for which the user specifies length, width, depth, thermal properties, and the number of nodes in the x and y directions as shown in Figure 3.

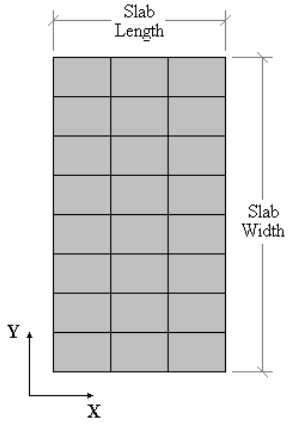


Figure 3: Slab Dimensions and Coordinate System

The slab is embedded in (surrounded on its sides and back by) soil referred to as the far-field. The far-field region is assumed to be an infinite energy sink/source (energy transfer with the far-field does not result in a temperature change of the far-field). The temperature of the far-field is governed by the Kasuda correlation.

The sizes of the slab nodes are automatically determined by the length and width of the slab and by the user specified number of nodes along the length and along the width of the slab. The slab is assumed to have a single node in the z direction.

3. Pipe Layout

Type712 allows users flexibility in describing how heating and/or cooling fluid is passed through the slab. Users are first asked to specify (as TRNSYS input file parameters) the number of pipes that pass through the slab and the number of fluid nodes into which the pipe should be broken. The isothermal nodes must be specified from the start of the pipe to the end of the pipe – along the direction of flow. The greater the number of nodes, the greater the accuracy – but at the cost of simulation speed. Users may specify multiple pipe nodes per slab node but by definition a pipe node cannot extend over multiple slab nodes. For each pipe, the user must provide an external data file that describes that pipe's path through the slab. The data file must contain the fraction of pipe length assigned to that pipe node and the coordinates of the slab node in which the pipe is embedded. Figure 4 shows an example in which a twelve-node pipe (uniform pipe nodal lengths in this example) passes through a twelve-node slab. In this example, the boundaries between pipe nodes correspond to the slab node boundaries.

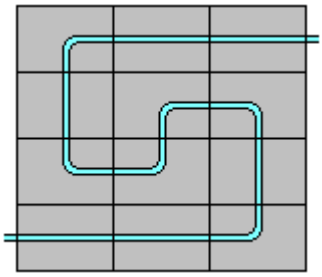


Figure 4: Example Pipe Layout with One Pipe Node Per Slab Node

The data file describing this pipe path is shown in Figure 5. The data file contains as many rows as there are nodes in the pipe. In each row, the first value is the fraction of the total pipe length assigned to the pipe node followed by the x and y coordinates of the slab node containing the pipe node.

0.0833	1,1
0.0833	2,1
0.0833	3,1
0.0833	3,2
0.0833	3,3
0.0833	2,3
0.0833	2,2
0.0833	1,2
0.0833	1,3
0.0833	1,4
0.0833	2,4
0.0833	3,4

Figure 5: Data File Describing Figure 4 Pipe Path

Figure 6 shows a slightly more complex example in which two pipes pass through a single slab. This second example is also meant to illustrate the method for specifying multiple pipe nodes per slab node

(the perimeter pipe in Figure 6) and the method for defining different lengths of pipe contained in a single slab node (the interior pipe in Figure 6).

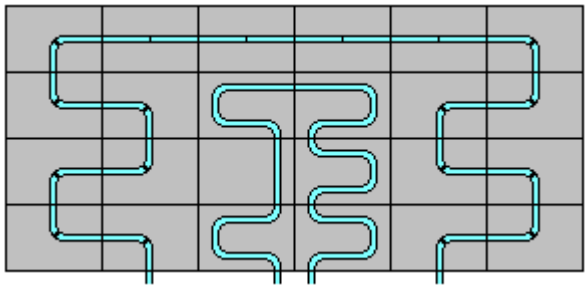


Figure 6: Slab Containing Two Pipe Paths

Data File Describing Flow Path for Perimeter Pipe	
0.0277	5,1
0.0277	5,1
0.0277	6,1
0.0277	6,1
0.0277	6,2
0.0277	6,2
0.0277	5,2
0.0277	5,2
0.0277	5,3
0.0277	5,3
0.0277	6,3
0.0277	6,3
0.0277	6,4
0.0277	6,4
0.0277	5,4
0.0277	5,4
0.0277	4,4
0.0277	4,4
0.0277	3,4
0.0277	3,4
0.0277	2,4
0.0277	2,4
0.0277	1,4
0.0277	1,4
0.0277	1,3
0.0277	1,3
0.0277	2,3
0.0277	2,3
0.0277	2,2
0.0277	2,2
0.0277	1,2
0.0277	1,2
0.0277	1,1
0.0277	1,1
0.0277	2,1
0.0277	2,1

Data File Describing Flow Path for Interior Pipe	
0.182	3,1
0.091	3,2
0.182	3,3
0.182	4,3
0.182	4,2
0.182	4,1

Figure 7: Data Files for Figure 6 Slab

It is worth noting that the user may specify there to be more than one pipe in a given slab node. However, these pipes will not "talk" to each other directly due to a temperature difference between them. They will each affect the same slab node, however, and so affect each other indirectly.

4. Energy Balances

The basic energy balance on a fluid node or slab node is shown in Equation 712.1:

$$mC_p \frac{dT}{dt} = \dot{Q}_{in} - \dot{Q}_{out} \quad \text{(Eq. 712.1)}$$

The equations describing Q_{in} and Q_{out} change depending upon the type of node and its location. Three types of nodes exist: fluid nodes, boundary nodes (between the slab and the far-field for example), and Type56 interface nodes.

4.1 Fluid Nodes

It is assumed that energy enters and leaves a fluid node through fluid flow into that section of pipe and through heat transfer with the slab. Conduction due to a temperature difference between fluid nodes is not considered. Thus when flow ceases, the fluid tends toward a steady state temperature only through conduction with the surrounding slab nodes and not through fluid conduction along its length. The appropriate Q_{in} term for flow is then:

$$\dot{Q}_{in} = \dot{m}C_p\left(T_{fluid,i-1} - T_{fluid,i}\right) \quad \text{(Eq. 712.2)}$$

Energy is transferred from the pipe node to the surrounding slab node via conduction according to Equation 712.3 where $\overline{T}$ denotes the average slab temperature over the time step.

$$\dot{Q}_{out} = UA\left(T_{fluid,i} - \overline{T}_{x,y}\right) \quad \text{(Eq. 712.3)}$$

It should be mentioned that the heat transfer out of the pipe node is then by definition the heat transfer into the slab node.

The UA term in equation 712.3 is made up of three parts; an internal convection resistance from the fluid to the pipe, a pipe wall resistance, and a conductive resistance in the slab.

The internal heat transfer resistance is simply:

$$R_{in} = \frac{1}{h_{inside} A_{surface,inner}} \quad \text{(Eq. 712.4)}$$

where:

$$h_{inside} = \frac{Nusselt\ k}{d_{pipe,i}} \quad \text{(Eq. 712.5)}$$

and for laminar flow (Reynolds number < 2300):

$$Nusselt = 4.36 \quad \text{(Eq. 712.6)}$$

and for turbulent flow (Reynolds number ≥ 2300):

$$Nusselt = 0.023\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{(1/3)} \qquad \text{(Eq. 712.7)}$$

The Reynolds number and Prandtl numbers are based on the inside diameter of the pipe and the fluid properties. The pipe wall resistance is defined by the radial conduction equation:

$$R_{wall} = \frac{\ln\left(\frac{d_{pipe,o}}{d_{pipe,i}}\right)}{2\,\pi\,k\,l} \qquad \text{(Eq. 712.8)}$$

In order to determine the conduction resistance from the pipe outer surface to the slab node, the conduction path length must be specified. In the case of a radiant floor, it is difficult to know this length since the exact path that the pipe follows through the slab node is not known. Instead, a contact resistance ($R_{contact}$) that describes the conductance between the pipe and the surrounding slab node is requested of the user as a parameter. This parameter can be thought of as a tuning parameter; used to adjust the heat transfer to measured data (if available).

The expression for the overall U value is:

$$U = \frac{1}{R_{inside} + R_{pipe} + R_{contact}} \qquad \text{(Eq. 712.9)}$$

4.2 Internal Slab Nodes

If a node does not have any pipes passing through it, its energy balance is made up of purely conductive terms. The equations for Q_{in} and Q_{out} are:

$$\dot{Q}_{in} = \frac{k_{soil}\,dy\,dz}{x_i - x_{i-1}}\left(\overline{T}_{x-1,y} - T_{x,y}\right) + \frac{k_{soil}\,dx\,dz}{y_i - y_{i-1}}\left(\overline{T}_{x,y-1} - T_{x,y}\right) \qquad \text{(Eq. 712.10)}$$

$$\dot{Q}_{out} = \frac{k_{soil}\,dy\,dz}{x_{i+1} - x_i}\left(T_{x,y} - \overline{T}_{x+1,y}\right) + \frac{k_{soil}\,dx\,dz}{y_{i+1} - y_i}\left(T_{x,y} - \overline{T}_{x,y+1}\right) \qquad \text{(Eq. 712.11)}$$

It should be noted that some internal soil nodes may have an additional resistance term added because Type712 allows insulation (specified as a purely resistive term) on the perimeter of the slab to extend down below the slab as far as desired.

4.3 Boundary Nodes

Nodes on the boundary between the slab and far-field are also governed by conduction but may have an additional pure resistance built into their energy balance. The resistive term is used to account for insulation around the perimeter of or beneath the slab.

In the examples above, the slab thermal conductivity and soil thermal conductivity are assumed equal. If this is not the case (which it usually isn't), the $\dot{Q}_{in}$ and $\dot{Q}_{out}$ terms are slightly modified to account for

the change in thermal conductivity due to the two materials. Refer to the source code for the governing equations if further clarification is needed.

The conduction length between the slab and the far-field is one-half of the corresponding nodal distance. For example, the conduction length between a node along the bottom of the slab and the far-field is dz/2.

4.4 Type56 Interface Nodes

When computing the effects of a boundary wall, Type56 requires, as an input, the temperature of the non-zone surface of the wall. One of the Type56 outputs is the temperature of the zone side surface of the same wall. When using Type712, the user has defined a fairly thin boundary wall inside Prebid. Referring again to Figure 1, this boundary wall is the hatch marked layer on top of the slab. Type56 computes the zone side surface temperature and passes it to Type712. Type712 in turn computes the temperature at the interface between the hatch marked layer and the slab node. An energy balance is written for the interface surface between the Type56 layer and the Type712 layer as shown in equation 712.12.

$$0 = UA_{\text{node}}\left(T_{\text{Type56}} - T_{\text{int}\,erface}\right) - kA_{node}\left(T_{\text{int}\,erface} - T_{SlabNode}\right) \qquad \text{(Eq. 712.12)}$$

A few points are worth bearing in mind. First, Type56 does not break the interior surface into nodes as is done with the Type712 slab; the Type56 surface has a single temperature. Consequently it is the area weighted average interface temperature that is delivered by Type712 to Type56.

Second, the U value in equation 712.14 is the overall U value for the slab layer contained within Type56. It can be found by generating the Type56 files from within Prebid and checking the contents of the resulting .inf file.

5. Solving the Resulting Equations

In this type of system, heat is transferred from a fluid in a pipe into an adjacent soil/slab node and then this heat is conducted to other soil nodes or to the interface between the slab material containing fluid pipes and the slab material in contact with zone air (modeled by Type56). This is not an easy problem to solve and compounding the problem is the fact that the effects of the pipe fluid mass must be considered for these types of systems. The problem breaks down into the required solution of two coupled differential equations, each having the form of equation 712.1; one for each slab node and one for each pipe node. The equations are coupled in that the pipe nodal equations depend on the pipe fluid temperatures and the pipe nodal equations depend on the slab nodal temperatures. While there are other available methods to solve coupled differential equations, Type712 solves the problem with an approximate analytical solution. The analytical solution has several inherent advantages over numerical solutions. First, the Type subroutine solves its own mathematical problem and does not rely on numerical recipes that must be attached to the subroutine externally. In this way, the subroutine can be imported into any FORTRAN compiler without problems. Secondly, some of the solution methods (mainly the numerical solutions) are dependent on the simulation time step and may not converge under certain circumstances commonly encountered in these types of systems (namely high flow rates for example). The analytical solution is time step independent but does require an iterative solution to be contained inside the Type subroutine to solve the coupled differential equations. While solving two

coupled differential equations iteratively can sometimes lead to convergence problems, this does not seem to be the case with this model under all operating scenarios tested.

To solve the differential equations analytically, each of the nodal the equations are placed into the form:

$$\frac{dT}{dt} = aT + b \qquad \text{(Eq. 712.13)}$$

where T is the dependent variable (temperature in this case), t is time, a is a constant and b may be a function of time or the dependent variable. If b is a constant, then the solution of this differential equation can be readily solved. If b is not constant, then a reasonable approximation to the analytical solution can be found by assuming that b is constant over the time step and equal to its average value over the time step.

At any time (for **a** not equal to zero):

$$T_{final} = (T_{initial} + \frac{\overline{b}}{a})\ e^{a\Delta t} - \frac{\overline{b}}{a} \qquad \text{(Eq. 712.14)}$$

where

$$\overline{b} = b(\overline{T}) \qquad \text{(Eq. 712.15)}$$

and

$$\overline{T} = \frac{1}{a\,\Delta t}\left(T_{initial} + \frac{\overline{b}}{a}\right)\left(e^{a\,\Delta t} - 1\right) - \frac{\overline{b}}{a} \qquad \text{(Eq. 712.16)}$$

With this assumption, the problem becomes straightforward to solve. The differential equation for each node is written in the correct form, a and $\overline{b}$ are determined for each node and T_{final} and T_{ave} are solved for . The value of $\overline{b}$ is then recalculated for each node and the process iterates until all temperatures converge.

While the assumption that b is constant over the time step (and equal to its average value) is not technically correct in our case (b for the pipe node is a function of the soil/slab temperature for example), it is a reasonable approximation for the small time steps we are using in the TRNSYS simulation (maximum time step=1 hour). The b terms hold the temperatures of other soil/slab nodes and pipe nodes. These temperatures in the b term are assumed to be constant for the solution of the nodal differential equations at their average value over the time step.

References

[1] Kasuda, T., and Archenbach, P.R. "Earth Temperature and Thermal Diffusivity at Selected Stations in the United States", ASHRAE Transactions, Vol. 71, Part 1, 1965